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Estimating Population Size from Commercial Statistics when Fishing Mortality varies with Age

by

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Introduction

A fish stock, if it is to maintain itself for any length of time, must reach an equilibrium in which the number of deaths each year is equal to the number of recruits. This suggests, therefore, that if total deaths can be measured, the annual recruitment and hence indirectly, the size of a stock can be estimated. For this purpose, particular interest is attached to those populations that are heavily exploited. The more heavily they are exploited, the more likely it is that most of the mortality is due to fishing and hence, since total catches can be counted more accurately, the better should one be able to estimate annual recruitment. More generally this method should be applicable to any stock in which one can determine

a) the total deaths due to fishing, and

b) the fraction of the total deaths due to all causes that this represents.

Of these, (a) can be determined from fishery statistics and by market sampling and (b) can be determined from the ratio of the instantaneous fishing mortality rate to instantaneous total mortality rate. Examples are given by Fry (1949) and Paloheimo (1958) but in this paper the method will be apllied to the situation in which the fishing mortality rate varies with age and, to provide a realistic numerical example, it has been applied to the estimation of the size of part of the North Sea whiting stock.

Estimating mortality rates from commercial statistics

The simplest procedure for estimating a total mortality rate from commercial catch data is to compare the catch per unit effort of a year-class on one occasion with its catch per unit effort on another occasion either twelve months earlier or later. The proportion surviving from one year to the next can then be calculated directly from the ratio of these catches per unit effort, and from this an instantaneous total mortality coefficient can be obtained. This is quite straightforward provided one can assume that the catches per unit effort provide comparable measures of year-class abundance. If this assumption does not hold, as it cannot if the fishing mortality rate varies with age, then a mortality rate calculated in this manner will be biased and in order to allow for this, a less direct procedure is necessary.

Let S_t be a true survival rate between the ages t and t+1. This will be a function of the natural mortality rate (M) and the mean fishing mortality rate between these ages

As a first approximation

$$S_t = \exp - \left[\tfrac{1}{2} (F_t + F_{t+1}) + M \right]$$

where F_t and F_{t+1} are the fishing mortality rates at ages t and t+1 respectively.

In the simplest case, where notional effort remains constant, the catches per unit effort at ages t and t+1 will be proportional to F_t and F_{t+1} respectively, i.e. the apparent survival rate will be

$$S'_t = \frac{F_{t+1}}{F_t} \cdot S_t$$

or

$$S'_t = \frac{F_{t+1}}{F_t} \exp-\left[\tfrac{1}{2}(F_t + F_{t+1}) + M\right]$$

This means that for one apparent survival rate there is an expression with three unknowns (F_t, F_{t+1}, and M). With two apparent survival rates there will be four unknowns, with three there will be five unknowns and so on. With two more unknowns than the number of observations no general solution is possible. However, one unknown can be eliminated by assuming a value for M, the natural mortality rate. A second can be eliminated by assuming a value for F at the oldest age concerned. Rather conveniently, it is found that estimates of F at the younger ages tend to be independent of the choice of F at the oldest age and so the solutions obtained are really only dependent on one's choice of M.

Further theoretical details are given by Jones (1961) where a formula is derived for determining a value of F_t, given a value F_{t+1} at the next higher age. The formula is

$$\tfrac{1}{2}F_t + \log_e F_t = Z'_t + \log_e F_{t+1} - \tfrac{1}{2}F_{t+1} - M \text{ -----: } (1)$$

where Z'_t is the apparent total instantaneous mortality rate from age t to age t+1

$$\text{i.e. } Z'_t = -\log_e S'_t$$

A species in which mortality varies with age appears to be the whiting in the North Sea, as shown by the mortality estimates obtained by Gambell (Table 1). The occurrence of negative mortality rates between age one and two and especially between two and three clearly suggests, in this instance, that young whiting are not fully represented in the landings and hence that the fishing mortality at these ages is less than that of the older fish.

Using formula (1) corrected mortality rates have been estimated from the apparent values given in Table 1. For the first trial, F at age 7 has been taken quite arbitrarily as 1.2o, and M has been taken as o.2 at all **ages**. A worksheet, setting out the calculation in detail is given in Table 2. Each row in this table shows the values used in a single application of equation (1). The first row, for example, shows the calculation of F_6, the fishing mortality rate at age 6, starting with an arbitrarily chosen value of 1.2o for F_7. The values of Z'_t have been taken from Table 1. The values of Q_t represent the right hand side of equation (1), using a value for M, of o.2 at all ages. Thus, for a value of F_7 equal to 1.2o, Q_6 equals o.672 and F_6 is found by trial and error to equal 1.12. This value is then entered in the column F_{t+1} for the second application of equation (1) in order to determine F_5 and so on. (All tables attached).

The values of F derived in Table 2 have been entered in Table 3 along with the results obtained by starting with values of F_7 equal to 1.4o and 1.oo. It would appear that the choice of F_7 within this range is not critical for the determination of the values of F between ages one and five. Furthermore, since the proportion of six year and older whiting in the landings is rather small, errors in the values of F_6 and F_7 are not important when it comes to estimating the total number of whiting of all ages. It is not critical therefore which set of mortality coefficients in Table 3 is used in subsequent calculations.

Calculation of F/Z at each age

Given values of F and M at each age it is a simple matter to estimate the ratios F/Z at each age. This has been done in Table 4, using the values of F determined in the third row of Table 4. It appears that for a value of M = o.2, about 8o% of whiting aged 3 years or more die as a result of fishing. At age two, just under half die from this cause and at age one only about 1/4oth.

Given values of $F/_Z$ at each age these can be used to raise estimates of numbers landed at each age to total deaths at each age. In this particular example, this can be done for whiting aged 3 years or more with some expectation of obtaining reasonably reliable estimates. For younger fish, and especially those aged one year, the likelihood of obtaining a reliable answer is rather low. In the case of the one year old fish, it would be necessary to multiply the number landed by 4o to obtain an estimate of the total one year old fish that die in a year. This is very unsatisfactory, not only because of possible errors in the value "4o", but because of uncertainty regarding the actual numbers of one year old whiting caught in the North Sea by vessels fishing for other than marketable demersal fish.

To circumvent this difficulty, the calculations have been completed for three year and older whiting only. This illustrates the method and at the same time provides an estimate of the abundance of whiting three years or more in ages that should be of the correct order of magnitude .

Determination of numbers landed

An estimate of the total number of marketable whiting landed from the North Sea is made in the report of the Lienesch Committee (196o). It is estimated there that total landings amounting to 71,ooo tons of whiting are equivalent to 28o million fish. Over the period 1955-59, North Sea whiting landings averaged 78,ooo tons annually and if this value is used, the equivalent number landed is 3o8 million.

Determination of numbers landed of each age

As a first approximation, the total landings of 3o8 million whiting can be broken down into age groups using market age composition data for commercial landings. Table 5, for example, shows the percentage age composition of whiting landed from the North Sea by Aberdeen trawlers in 196o. These data are not wholly suitable, however, as brood fluctuations are liable to distort the picture. The percentage of five year olds appears to be unusually high, for example, due to the fact that the 1955 year-class was an unusually good one. To circumvent this difficulty, an age composition independent of brood fluctuations can be constructed using the data in Table 1. Since the apparent values of Z in this table reflect changes in the age composition of the landings, they can be used directly to generate an idealised age composition. Thus, starting for convenience at age 2, with an arbitrary number of fish equal to loo one gets the number at age one as loo $e^{-3.17} = 4$, and the number at age three at loo $e^{0.87} = 239$. The number at age 4 becomes

$$239\ e^{-0.64} = 126, \text{ and so on.}$$

These values are entered in Table 6, along with the same series on a percentage basis. This percentage age composition, which as it happens is not so different from the percentage age composition given in Table 5 can be used to break down the overall value of 3o8 million fish into age groups, and this has been done in the second row of Table 7.

Determination of the number of whiting in the North Sea

The steps in the calculation of the total deaths in a year are given in Table 7. The first row shows the numbers landed. The second row shows the ratios $Z/_F$, rather than $F/_Z$, and the third row is obtained by multiplying the numbers landed by the corresponding values of $Z/_F$. These values, summed from age 3 give an estimate of the total deaths of three years and older whiting in a year. This is equal to 311 million fish.

This then is an estimate of the number in a year-class at an age of three years. More precisely the age referred to can be determined by consideration, in the first instance, of the mortality rates shown in Table 1. These are shown as though they refer to the ages 1-2, 2-3 and so on. In fact, each was calculated over a range of annual periods such that they refer to ages 1.2-2.2, 2.2-3.2 and so on. It follows therefore that the values of F_t in Table 3 refer to the ages 1.2, 2.2 etc. and that the age frequencies in Table 6 are calculated at the ages 1.2, 2.2 etc. So-called 1+ fish in Tables 6 and 7 therefore range from o.7-1.7 years. 2+ fish range from 1.7-2.7 and so on. The age at which the value of 311 million recruits has been calculated is therefore 2.7 years.

Finally a measure of the number in the sea can also be determined by summing the number of deaths to various ages. For example, an average of (36 + 7 + 4) = 47 million fish aged 5 years and older die each year. There must therefore be at least 47 million fish in the sea at an age of 4.7 years if this is to happen. The numbers in the sea at the beginning of each age can be arrived at in this way and the values are shown in the last row of Table 7.

If these, in their turn are summed, the value of 5o7 million is reached for the average number of whiting in the North Sea aged 2.7 years and more.

Finally it is necessary to emphasize that the calculations set out in this paper have been performed with the object of demonstrating a particular method of calculation, rather than of estimating the number of whiting in the North Sea. Nevertheless it seems reasonable to suppose that the estimates given for whiting aged 2.7 years and more are of the correct order of magnitude. No attempt however has been made to estimate the even larger numbers of whiting younger than this. To do this it would be necessary to take account of the numbers of young whiting taken in "mixed fisheries" and also of the numbers rejected by the "Convention fisheries". For estimates of these numbers the reader is referred to the report of the Lienesch Committee (196o).

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Abstract

One method of estimating population size is based on the fact that when a stock is in equilibrium, the number dying each year must equal the number of recruits. In the case of a commercially exploited stock the numbers that die each year can be determined from the annual catch if it is possible to measure what fraction of the total deaths this represents. This fraction is in fact equivalent to the ratio of the instantaneous fishing mortality rate to the instantaneous total mortality rate. In this paper a method is described of estimating this when fishing mortality varies with age and, for illustration, is applied to the estimation of the size of part of the North Sea whiting stock.

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Table 1. Showing the apparent values of Z (Z'_t) for North Sea whiting (from Gambell)

Age	Z'_t
1-2	-3.17
2-3	- .87
3-4	+ .64
4-5	.86
5-6	1.62
6-7	1.29

Table 2. Worksheet, showing successive steps in the calculation of F_t at various ages

t+1	F_{t+1}	$\log_e F_{t+1}$	$\frac{1}{2} F_{t+1}$	t	Z'_t	Q_t	F_t ⁎
7	1.2o	o.182	o.6oo	6	1.29	o.672	1.12
6	1.12	o.113	o.56o	5	1.62	o.973	1.35
5	1.35	o.3oo	o.675	4	o.86	o.285	o.86
4	o.86	-o.151	o.43o	3	o.64	-o.141	o.63
3	o.63	-o.462	o.315	2	-o.87	-1.847	o.15
2	o.15	-1.897	o.o75	1	-3.17	-5.342	o.oo5

$$Q_t = Z'_t + \log_e F_{t+1} - \frac{1}{2} F_{t+1} - 0.2$$

⁎ F_t satisfies the relationship $\frac{1}{2} F_t + \log_e F_t = Q_t$

Table 3. Showing three trial sets of estimates of the fishing mortality rate (F) for North Sea whiting

Trial \ F	F_1	F_2	F_3	F_4	F_5	F_6	F_7
1	o.oo5	o.15	o.63	o.86	1.35	1.12	1.2o
2	o.oo5	o.15	o.63	o.86	1.36	1.16	1.4o
3	o.oo5	o.15	o.63	o.86	1.33	1.o6	1.oo

It is assumed that the natural mortality rate is o.2 at all ages.

Table 4. Showing estimated values of F, Z and F/Z for North Sea whiting

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
F	o.oo5	o.15	o.63	o.86	1.33	1.o6	1.oo
Z	o.2o5	o.35	o.83	1.o6	1.53	1.26	1.2o
F/Z	o.o24	o.43	o.76	o.81	o.87	o.84	o.83

Table 5. Showing the percentage age composition of whiting landed from the northern North Sea by Aberdeen steam trawlers in 196o

Year-class	Age	%
1959	1+	1
1958	2+	21
1957	3+	44
1956	4+	4
1955	5+	24
1954	6+	3
1953	7+	1
1952 and older	8+ and older	3

Table 6. Showing the mean age composition of North Sea whiting, calculated using the values of Z in Table 1

Age	Arbitrary numbers	%
1+	4	1
2+	1oo	19
3+	239	44
4+	126	23
5+	53	1o
6+	1o	2
7+	3	1

Table 7. Showing the steps in the calculation of the total numbers (millions) of 3 years and older whiting in the North Sea

Age	1+	2+	3+	4+	5+	6+	7+	Total
Number landed	3 x	59	135	71	31	6	3	3o8
Z/F	41 x	2.33	1.31	1.23	1.15	1.19	1.2o	-
Total deaths	-	-	177	87	36	7	4	311
Number in sea	-	-	311	134	47	11	4	5o7

x Unreliable values